Variabili casuali
medie, varianza e covarianza, correlazione

Variabile casuale semplice $X = \square_{k=1}^{n} x_k$ **ponderata** $(X,P) = \square_{k=1}^{n} (x_k, p_k)$

- **Media aritmetica**
$$m_1(X) = \frac{\sum_{k=1}^{n} x_k}{n} \qquad m_1(X,P) = \frac{\sum_{k=1}^{n} x_k p_k}{\sum_{k=1}^{n} p_k}$$

- **Media armonica**
$$m_{-1}(X) = \frac{n}{\sum_{k=1}^{n} \frac{1}{x_k}} \qquad m_{-1}(X,P) = \frac{\sum_{k=1}^{n} p_k}{\sum_{k=1}^{n} \frac{p_k}{x_k}}$$

- **Media geometrica**
$$m_0(X) = \sqrt[n]{\prod_{k=1}^{n} x_k} \qquad m_0(X,P) = \left(\prod_{k=1}^{n} x_k^{p_k}\right)^{\frac{1}{\sum_{k=1}^{n} p_k}}$$

- **Media quadratica**
$$m_2(X) = \sqrt{\frac{\sum_{k=1}^{n} x_k^2}{n}} \qquad m_2(X,P) = \sqrt{\frac{\sum_{k=1}^{n} x_k^2 p_k}{\sum_{k=1}^{n} p_k}}$$

- **Media potenziale (ordine r)**
$$m_r(X) = \sqrt[r]{\frac{\sum_{k=1}^{n} x_k^r}{n}} \qquad m_r(X,P) = \sqrt[r]{\frac{\sum_{k=1}^{n} x_k^r p_k}{\sum_{k=1}^{n} p_k}}$$

- **Varianza**
$$\sigma_X^2 = \frac{\sum_{k=1}^{n} (x_k - m_1(X))^2}{n} = m_2(X) - (m_1(X))^2$$
$$\sigma_{(X,P)}^2 = \frac{\sum_{k=1}^{n} (x_k - m_1(X))^2 p_k}{\sum_{k=1}^{n} p_k} = m_2(X,P) - (m_1(X,P))^2$$

Variabile casuale doppia $(X,Y) = \square_{k=1}^{n} (x_k, y_k)$ … $(X,Y,P) = \square_{k=1}^{n} (x_k, y_k, p_k)$

- **Momento secondo misto**
$$m_2(X,Y) = \frac{\sum_{k=1}^{n} x_k y_k}{n} \qquad m_2(X,Y,P) = \frac{\sum_{k=1}^{n} x_k y_k p_k}{\sum_{k=1}^{n} p_k}$$

- ***Covarianza***

$$\sigma_{(X,Y)} = \frac{\sum_{k=1}^{n}(x_k - m_1(X))(y_k - m_1(Y))}{n} = m_2(X,Y) - m_1(X)\cdot m_1(Y)$$

$$\sigma_{(X,Y,P)} = \frac{\sum_{k=1}^{n}(x_k - m_1(X))(y_k - m_1(Y))\,p_k}{\sum_{k=1}^{n} p_k} = m_2(X,Y,P) - m_1(X,P)\cdot m_1(Y,P)$$

- ***coefficiente di correlazione lineare di Bravais - Pearson***

$$\rho_{(X,Y)} = \frac{\sigma_{(X,Y)}}{\sqrt{\sigma_X^2\,\sigma_Y^2}} = \frac{\sum_{k=1}^{n}(x_k - m_1(X))(y_k - m_1(Y))}{\sqrt{\left(\sum_{k=1}^{n}(x_k - m_1(X))^2\right)\cdot\left(\sum_{k=1}^{n}(y_k - m_1(Y))^2\right)}} = \frac{m_2(X,Y) - m_1(X)\cdot m_1(Y)}{\sqrt{(m_2(X) - (m_1(X))^2)\cdot(m_2(Y) - (m_1(Y))^2)}}$$

$$\rho_{(X,Y,P)} = \frac{\sigma_{(X,Y,P)}}{\sqrt{\sigma_{(X,P)}^2\,\sigma_{(Y,P)}^2}} = \frac{\sum_{k=1}^{n}(x_k - m_1(X))(y_k - m_1(Y))\,p_k}{\sqrt{\left(\sum_{k=1}^{n}(x_k - m_1(X))^2\,p_k\right)\cdot\left(\sum_{k=1}^{n}(y_k - m_1(Y))^2\,p_k\right)}} = \frac{m_2(X,Y,P) - m_1(X,P)\cdot m_1(Y,P)}{\sqrt{(m_2(X,P) - (m_1(X,P))^2)\cdot(m_2(Y,P) - (m_1(Y,P))^2)}}$$

- ***coefficiente di regressione della variabile Y rispetto alla variabile X***

$$\beta_{(X,Y)} = \frac{\sigma_{(X,Y)}}{\sigma_X^2} = \frac{\sum_{k=1}^{n}(x_k - m_1(X))(y_k - m_1(Y))}{\sum_{k=1}^{n}(x_k - m_1(X))^2} = \frac{m_2(X,Y) - m_1(X)\cdot m_1(Y)}{m_2(X) - (m_1(X))^2}$$

$$\beta_{(X,Y,P)} = \frac{\sigma_{(X,Y,P)}}{\sigma_{(X,P)}^2} = \frac{\sum_{k=1}^{n}(x_k - m_1(X))(y_k - m_1(Y))\,p_k}{\sum_{k=1}^{n}(x_k - m_1(X))^2\,p_k} = \frac{m_2(X,Y,P) - m_1(X,P)\cdot m_1(Y,P)}{m_2(X,P) - (m_1(X,P))^2}$$

$$\rho_{(X,Y)} = \frac{\sigma_{(X,Y)}}{\sqrt{\sigma_X^2\,\sigma_y^2}} = \sqrt{\frac{\sigma_{(X,Y)}^2}{\sigma_X^2\,\sigma_y^2}}\cdot sgn(\sigma_{(X,Y)}) = \sqrt{\frac{\sigma_{(X,Y)}}{\sigma_X^2}\cdot\frac{\sigma_{(X,Y)}}{\sigma_y^2}}\cdot sgn(\sigma_{(X,Y)}) = \sqrt{\beta_{(Y,X)}\cdot\beta_{(X,Y)}}\cdot sgn(\sigma_{(X,Y)})$$